

# Sensorless speed control including zero speed of non salient PM synchronous drives

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**Abstract.** This paper presents a position sensorless drive of non salient pole PM synchronous motors for all speeds including zero speed. Using adaptive Lyapunov design a new approach for the design of an observer is developed. The resulting scheme leads to a nonlinear full order observer for the motor states including the rotor speed. Assuming motor parameters known the design achieves stability with guaranteed region of attraction even at zero speed. The control method is made robust at zero and low speed by changing the direct vector current component to a value different from zero. In order to verify the applicability of the method the controller has been implemented and tested on a 800 W motor.

**Key words:** PM motors, low speed operation, speed sensorless control.

## Nomenclature:

$a$	complex spatial operator $e^{j2\pi/3}$
$i_{sA,B,C}$	stator phase currents A,B and C
$u_{sA,B,C}$	stator phase voltages A,B and C
$i_s$	stator current complex space vector
$u_s$	stator voltages complex space vector
$\psi_s$	stator flux
$\psi_r$	rotor flux
$R$	stator resistances
$L$	stator inductance
$\omega$	synchronous angular frequency
$\omega_{mech}$	rotor speed
$p$	time derivative operator $d/dt$
$Z_p$	number of pole pair
$\theta$	rotor position

## 1. Introduction

The permanent magnet synchronous motor (PMSM) offers high power density implying reduced size and less material to transport. PM motors cannot be operated in open loop due to the highly unstable behavior of the motor dynamics. This means that PM motors needs a measurement of the rotor position in order to control the motor in a robust way. The traditional method to determine the rotor position is to use an encoder or resolver, but these components are expensive and will add additional cost to the motor.

In more than ten years there has been an extensive research in finding reliable position sensorless methods to estimate the rotor position from the applied voltages and the consumed currents. The most significant papers of the research up to 1996 can be found in the reference [1]. In this paper focus will be on PMSM type of motor due to the fact that the results are mend for low noise emission applications like pumps for domestic use. In PMSM solutions the rotor position is normally

determined by an open loop or closed loop observer see [2] or by voltage injection methods exciting saliency or saturation effects in the motor see [3,5,6]. In [8] sensorless salient-pole PM synchronous motor drive in all speed ranges are obtained by switching between a back-emf (BEMF) method in the medium and high speed range and an injection method in the low and zero speed range.

For a non salient PM synchronous motor the injection methods described are not valid and no BEMF is presented at zero speed. This is normally handled by a start-up procedure operating the motor in open loop up to a given minimum speed where BEMF is reliable, and after this point a jump to observer based field-oriented control takes place. This jump can be noise full and can give extreme speed transients and pull out can in severe situations occur.

In this paper a new method based on Lyapunov stability will be presented operating from zero speed without changing the control structure. The idea of the method chosen is to force direct current into the machine in the faulty position the observer estimates at low speed; this will force the rotor position to the incorrectly estimated position, and the difference between the real position and the estimated position will be reduced.

The method is implemented and verified experimentally on a 800 W motor. The results demonstrate that the method works successfully from zero to full speed. The method is able to produce estimates of position and speed with a precision good enough to replace a shaft sensor.

## 2. Observer design

**2.1. Voltage equations.** In the rotor oriented coordinate system  $e^{j\theta}$  the voltage equation for the motor is

$$u_s^{(R)} = Ri_s^{(R)} + (p + j\omega)Li_s^{(R)} + j\omega\psi_M \quad (1)$$

with  $\omega = p\theta$ . Where  $p$  is the time derivative operator.

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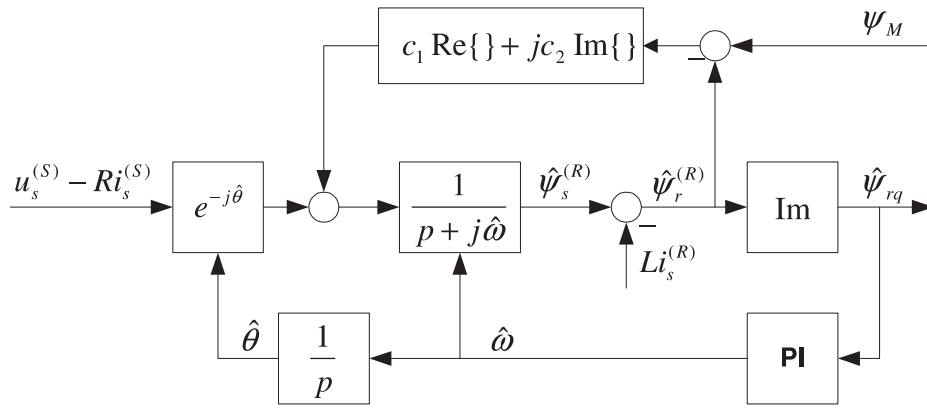


Fig. 1. Rotor angle observer

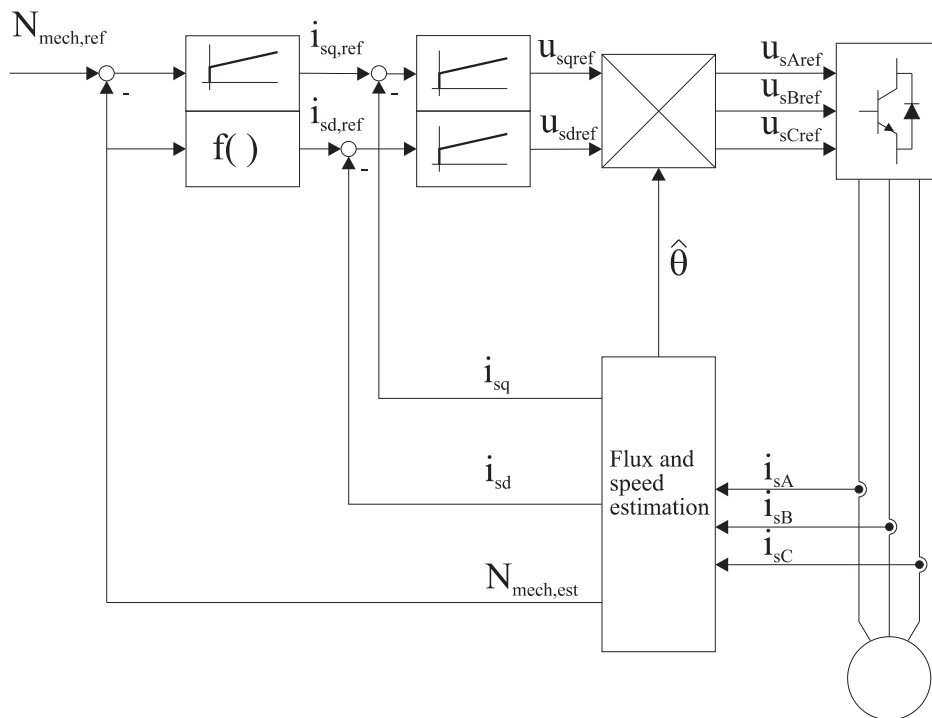


Fig. 2. Rotor field oriented control

In the coordinate system  $e^{j\hat{\theta}}$  of the estimated rotor angle  $\hat{\theta}$  the voltage equation is

$$u_s^{(\hat{R})} = Ri_s^{(\hat{R})} + (p + j\hat{\omega})Li_s^{(\hat{R})} + (p + j\hat{\omega})\psi_M e^{j(\theta - \hat{\theta})} \quad (2)$$

with  $\hat{\omega} = p\hat{\theta}$

Omitting  $(\hat{R})$  in the following and introducing  $\tilde{\theta} = \hat{\theta} - \theta$  (2) gives

$$u_s = Ri_s + (p + j\hat{\omega})Li_s + j\hat{\omega}\psi_M + (\omega \sin \tilde{\theta} + j(\omega \cos \tilde{\theta} - \hat{\omega}))\psi_M. \quad (3)$$

**2.2. Observer candidate.** From (3) the stator flux linkage is given by

$$\psi_M + Li_s = \frac{1}{p + j\hat{\omega}} \{ u - Ri_s - (\omega \sin \tilde{\theta} + j(\omega \cos \tilde{\theta} - \hat{\omega}))\psi_M \} \quad (4)$$

Because the term  $\{ \omega \sin \tilde{\theta} + j(\omega \cos \tilde{\theta} - \hat{\omega}) \} \psi_M$  is unknown a feedback observer may be given the form

$$\hat{\psi}_r + Li_s = \frac{1}{p + j\hat{\omega}} \{ u_s - Ri_s + v \} \quad (5)$$

where equations for  $v$  and  $\hat{\omega}$  have to be calculated.

From (4) and (5) the error dynamics are becomes

$$p(\hat{\psi}_r - \psi_M) = -j\hat{\omega}(\hat{\psi}_r - \psi_M) + v + (\omega \sin \tilde{\theta} + j(\omega \cos \tilde{\theta} - \hat{\omega}))\psi_M. \quad (6)$$

Stabilizing with  $v = -c_1(\hat{\psi}_{rd} - \psi_M) - c_2\hat{\psi}_{rq}$  and taking the

real and imaginary part of Eq. (6) gives

$$\begin{aligned} p(\hat{\psi}_{rd} - \psi_M) &= -c_1(\hat{\psi}_{rd} - \psi_M) + \hat{\omega}\hat{\psi}_{rq} \\ &\quad + \omega \sin \tilde{\theta} \psi_M \\ p\hat{\psi}_{rq} &= -c_2\hat{\psi}_{rq} - \hat{\omega}(\hat{\psi}_{rd} - \psi_M) \\ &\quad - (\hat{\omega} - \omega \cos \tilde{\theta})\psi_M \end{aligned} \quad (7)$$

$$\begin{aligned} p(\hat{\psi}_{rd} - \psi_M) &= -c_1(\hat{\psi}_{rd} - \psi_M) + \hat{\omega}\hat{\psi}_{rq} \\ &\quad - \omega\hat{\psi}_{rq} \\ p\hat{\psi}_{rq} &= -c_2\hat{\psi}_{rq} - \hat{\omega}(\hat{\psi}_{rd} - \psi_M) \\ &\quad - (\hat{\omega} - \omega)\psi_M + \omega(\hat{\psi}_{rd} - \psi_M) \end{aligned} \quad (8)$$

Insertion of  $\tilde{\omega} = \hat{\omega} - \omega$  in (8) now gives

$$\begin{aligned} p(\hat{\psi}_{rd} - \psi_M) &= -c_1(\hat{\psi}_{rd} - \psi_M) + \tilde{\omega}\hat{\psi}_{rq} \\ p\hat{\psi}_{rq} &= -c_2\hat{\psi}_{rq} - \tilde{\omega}(\hat{\psi}_{rd} - \psi_M) - \tilde{\omega}\psi_M. \end{aligned} \quad (9)$$

**2.3. Lyapunov analysis.** Stability of the error equations may be obtained by using a Lyapunov function candidate

$$P = \frac{1}{2} \left\{ (\hat{\psi}_{rd} - \psi_M)^2 + (\hat{\psi}_{rq})^2 + \frac{1}{\gamma} \tilde{\omega}^2 \right\}. \quad (10)$$

Using (9) in the time derivative of (10) gives

$$\frac{dP}{dt} = -c_1(\hat{\psi}_{rd} - \psi_M)^2 - c_2(\hat{\psi}_{rq})^2 + \tilde{\omega}(-\psi_M\hat{\psi}_{rq} + \frac{1}{\gamma} \frac{d\tilde{\omega}}{dt}). \quad (11)$$

Choosing  $\frac{d\tilde{\omega}}{dt} = \gamma\psi_M\hat{\psi}_{rq}$  in (11) then gives

$$\frac{dP}{dt} = -c_1(\hat{\psi}_{rd} - \psi_M)^2 - c_2(\hat{\psi}_{rq})^2. \quad (12)$$

For  $c_1 > 0$  and  $c_2 > 0$  the Lyapunov function candidate is shown to be a Lyapunov function and the following convergence is obtained

$$\hat{\psi}_r = \hat{\psi}_{rd} + j\hat{\psi}_{rq} \rightarrow \psi_M. \quad (13)$$

Because we have  $\hat{\psi}_{rq} = -\psi_M \sin \tilde{\theta}$  it is also shown that  $\hat{\theta} \rightarrow \theta$ .

Now we only have to examine the adaptation condition

$$\frac{d\tilde{\omega}}{dt} = \gamma\psi_M\hat{\psi}_{rq}. \quad (14)$$

If  $\omega$  is assumed constant the adaptation condition becomes

$$\frac{d\tilde{\omega}}{dt} = \gamma\psi_M\hat{\psi}_{rq}. \quad (15)$$

The assumption of constant  $\omega$  means in practice that the variation of  $\omega$  has to be slow compared to the adaptation time constant, which depends of  $\gamma$ .

With the above assumptions the following observer is obtained

$$\begin{aligned} \hat{\psi}_r &= \hat{\psi}_s - Li_s \\ \frac{d}{dt} \hat{\psi}_s &= u_s - Ri_s - c_1(\hat{\psi}_{rd} - \psi_M) - jc_2\hat{\psi}_{rq} - j\hat{\omega}\hat{\psi}_s \\ \frac{d}{dt} \hat{\omega} &= \gamma\psi_M\hat{\psi}_{rq} \\ \frac{d}{dt} \hat{\theta} &= \hat{\omega} \end{aligned} \quad (16)$$

For known values of the initial rotor position,  $(R, L, \psi_M)$ , no offset due to drift and perfect dead time compensation the observer converges to the correct rotor angle. Because this is not to be expected in practice a robustness analysis has to be performed.

**2.4. PI adjustment.** The adjustment of  $\tilde{\omega}$  given by the Lyapunov method is an integral controller

$$\tilde{\omega}(t) = \gamma\psi_M \int_{-\infty}^t \hat{\psi}_{rq}(\tau) d\tau. \quad (17)$$

It may be expected that a quicker adaptation can be achieved by using a PI controller

$$\tilde{\omega}(t) = \gamma_1\hat{\psi}_{rq}(t) + \gamma_2 \int_{-\infty}^t \hat{\psi}_{rq}(\tau) d\tau. \quad (18)$$

Since a system with the transfer function

$$H(s) = \gamma_1 + \frac{\gamma_2}{s} \quad (19)$$

is output strictly passive for positive  $\gamma_1$  and  $\gamma_2$  it follows from the passivity theorem [7], that PI adjustment is stable if a transfer function from  $-\tilde{\omega}$  to  $\hat{\psi}_{rq}$  is strictly positive real. From (9) a transfer function  $G(s)$  is found (20)

$$\hat{\psi}_{rq} = -\frac{p + c_1}{p^2 + (c_1 + c_2)p + c_1c_2 + \tilde{\omega}^2} \psi_M \tilde{\omega} = -G(p) \psi_M \tilde{\omega}. \quad (20)$$

For  $c_1 > 0$  and  $c_2 > 0$  the transfer function  $G(s)$  is strictly positive real because it has no poles in the right half-plane, no poles and zeros on the imaginary axis and

$$\text{Re}\{G(j\omega)\} = \frac{c_1(c_1c_2 + \tilde{\omega}^2) + c_2\omega^2}{\|(-\omega^2 + j\omega(c_1 + c_2) + c_1c_2 + \tilde{\omega}^2)\|} > 0.$$

The observer with PI adjustment given by (21) is shown in Fig. 1

$$\begin{aligned} \hat{\psi}_r &= \hat{\psi}_s - Li_s \\ \hat{\psi}_s &= \int^t (u_s - Ri_s - c_1(\hat{\psi}_{rd} - \psi_M) \\ &\quad - jc_2\hat{\psi}_{rq} - j\hat{\omega}\hat{\psi}_s) d\tau \\ \hat{\omega} &= \gamma_1\hat{\psi}_{rq} + \gamma_2 \int^t \hat{\psi}_{rq} d\tau \\ \hat{\theta} &= \int^t \hat{\omega} d\tau \end{aligned} \quad (21)$$

**2.5. Robustness.** Because the Lyapunov analysis has shown stability, the robustness is analyzed in steady state with  $\hat{\omega} = \omega$ . Let us assume that uncompensated dead time compensation and error in  $R$  gives and voltage error  $\delta u$ . The observer equations then gives

$$0 = \delta u + j\omega\psi_M - c_1(\hat{\psi}_{rd} - \psi_M) - jc_2\hat{\psi}_{rq} - j\omega\hat{\psi}_r. \quad (22)$$

For  $\|\omega\| \gg c_1$  and  $\|\omega\| \gg c_2$  the real and imaginary part of this equation gives

$$\begin{aligned} \hat{\psi}_{rd} - \psi_M &\approx \delta u_q / \omega \\ \hat{\psi}_{rq} &\approx -\delta u_d / \omega \end{aligned} \quad (23)$$

leading to

$$\sin(\hat{\theta} - \theta) \approx \frac{\delta u_d}{\omega\psi_M}. \quad (24)$$

Because these inequalities calls for small values of  $(c_1, c_2)$  and the dynamics of the observer calls for  $(c_1, c_2)$  as big as possible, the practical choice is a compromise between fast observer dynamics and small steady state estimation error. Experiments and simulations show that best performance is obtained for  $c_2 \ll c_1$  and that the performance is insensitive for the choice of the values. In the experiments shown the values  $(c_1, c_2) = (50, 1)$  are used.

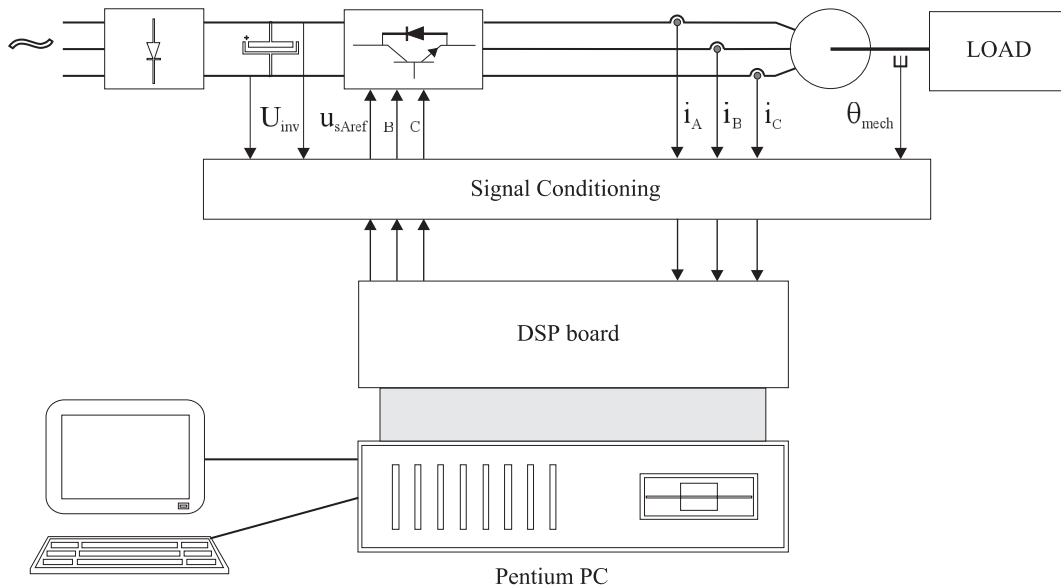


Fig. 3. Laboratory setup

**2.6. Zero and very low speed control.** For known values of the initial rotor position,  $(R, L, \psi_M)$ , no offset due to drift and perfect dead time compensation the observer converges to the correct rotor angle. Because this is not to be expected in practice the robustness analysis has shown that the observer is stable but gives large error of the estimated angle  $\hat{\theta}$  for small values of  $\|\omega\|$ . The idea of the method chosen is to force direct current into the machine in the faulty position the observer estimates at low speed; this will force the rotor position to the incorrectly estimated position, and the difference between the real position and the estimated position will be reduced.

Figure 2 shows the rotor field oriented control system. At zero and very low speed the reference value for  $i_{sd}$  is given a value different from zero. If the rotor angle is estimated correctly a value for  $i_{sd} \neq 0$  gives no torque. If the rotor angle is estimated with an error the rotor is forced in the direction of  $\hat{\theta}$  used in the controller. The error at zero speed depends on  $i_{sd,ref}$  due to the fact that we have  $m_e = 3/2 Z_p \psi_M i_{sd} \sin(\hat{\theta})$ .

This new principle means that no manual mode at zero and low speed is necessary, closed loop control is obtained for all values of the speed reference. The function for  $i_{sd,ref}$  is

$$i_{sd,ref} = i_{ref0} e^{-|\hat{\omega}_r|/\omega_0} \quad (25)$$

with  $i_{ref0}$  determined by the maximal load torque expected at zero speed and  $\omega_0$  small due to the fact that estimation error of the rotor angle is only significant at low speed.

### 3. Experiments

**3.1. Laboratory setup.** The laboratory setup shown in Fig. 3 is based on Real Time Workshop, Simulink and DSpace. The drive system is via a signal conditioner connected to a DSP board in the computer. The control software is Simulink blocks written in C. The nominal motor parameters are

$$\begin{array}{ccccccc} R & L & \psi_M & Z_p & N_{mech} & & \\ \text{Nom.} & 4.0 \Omega & 0.013 \text{ H} & 0.3 \text{ Vs} & 3 & 1500 \text{ rpm} & \end{array} \quad (26)$$

**3.2. Initial startup.** Figures 4a and 4b show the start-up response with the rotor angle initialized to the worst condition, which is to be opposite to the initial value given the estimate of the rotor position. The initial negative speed in Fig. 4a is due to the fact that the estimation error of the rotor angle  $\hat{\theta} - \theta > 90$  degrees. This situation only occurs the very first time the control system is started. Next time the reference is set to zero speed the angle is estimated with a small estimation error due to the  $i_{sd,ref}$  value different from zero.

**3.3. Rotor speed step responses.** Figure 4c shows a step response for a very low speed. The scale of the speed axis has to be noticed showing that the variation of the rotor speed is less than 5 rpm.

Figures 4d, 4e and 4f show step response for 0–500 rpm, 0–1000 rpm and 0–1500 rpm respectively. The curve form of the responses are as expected and the sound of the motor also indicate field orientation during the transients.

**3.4. Load step response.** Figure 5 shows the response of step in the load torque both at 0 rpm and at 1000 rpm.

Figures 5a and 5b show the speed variation for step in the load torque. Even at zero speed reference the speed is equal to zero in steady state.

Figures 5c and 5d show the  $i_{sd}$  and  $i_{sq}$  current. Figure 5d shows the normal response at 1000 rpm where  $i_{sd} = 0$  and  $i_{sq}$  is the torque producing current. Figure 5c shows the  $i_{sd}$  and  $i_{sq}$  at zero speed. The  $i_{sd}$  is given a reference value equal to 5A and when the load torque tries to turn the rotor axis the angle between this current in the estimated rotor angle and the real rotor angle produces a torque equal to the load torque and the speed is kept equal to zero as seen from Fig. 5a

Figures 5e and 5f show the error between the estimated and real rotor angle. The error at zero speed depends on  $i_{sd,ref}$  due to the fact that we have  $m_e = 3/2 Z_p \psi_M i_{sd} \sin(\hat{\theta})$ . Figure 5f shows the error at high speed and the scale of the axis has to be noticed.

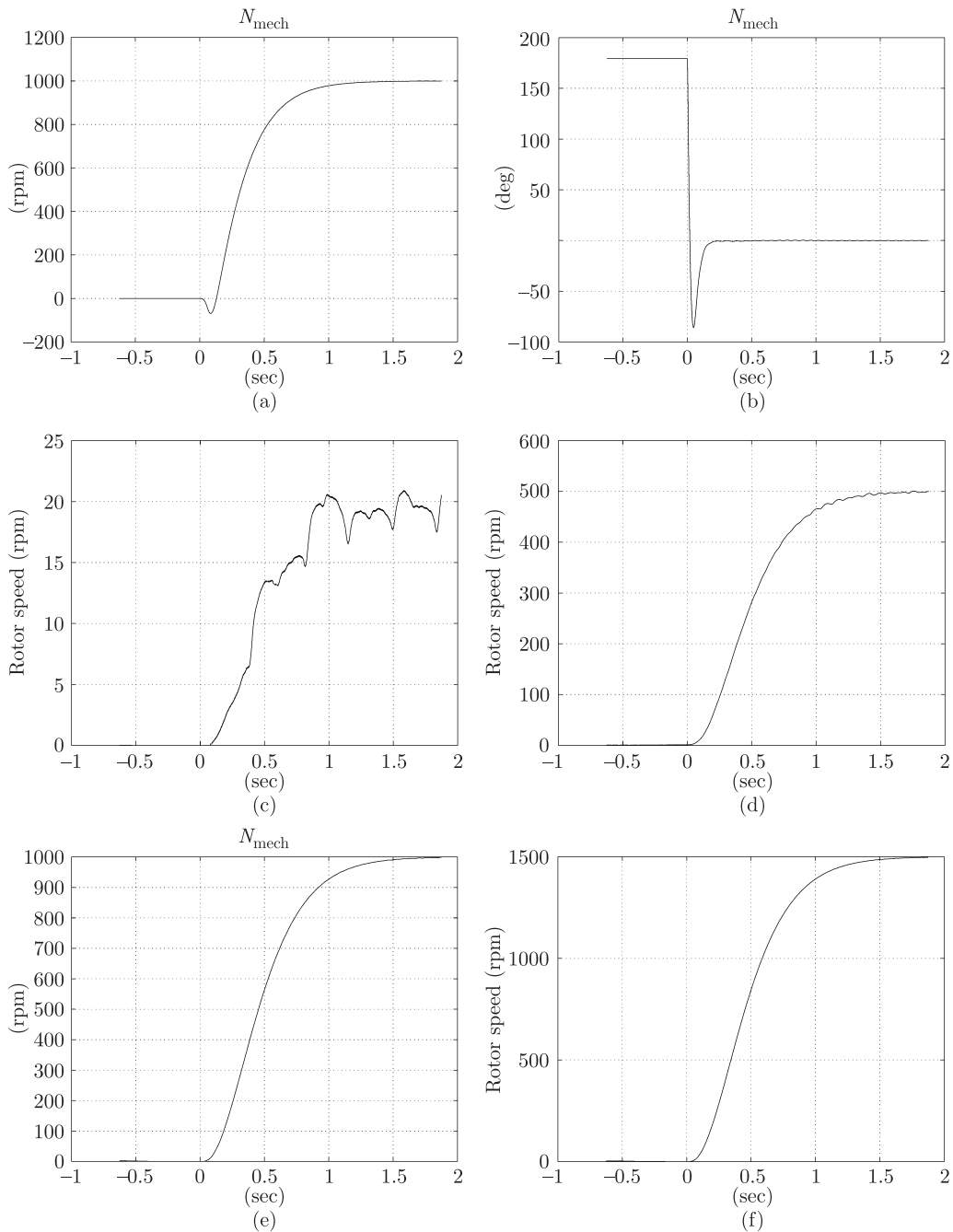


Fig. 4. Experimental waveforms of estimated speed (a,c,d,e,f) and rotor angle (b) during motor starting process: (a) first rotor speed step response with rotor angle initialized to an  $180^\circ$  error at startup, (b) convergence of the estimated rotor angle with rotor angle initialized to an  $180^\circ$  error at startup, (c) very low rotor speed step response 0–20 rpm, (d) low rotor speed step response 0–500 rpm, (e) rotor speed step response 0–1000 rpm, (f) high rotor speed step response 0–1500 rpm

#### 4. Conclusions

A new observer for the rotor angle is presented. Stability of the observer is proven by the Lyapunov method and robustness is analyzed in steady state. Various papers concerning methods for starting PMSM without position sensors have been presented. Most methods have a special mode for start-up and operations at low speeds. The proposed method operates in the same mode from zero speed to maximum speed, which simplifies the control algorithm and eliminates the lag of robustness

when a controller shifts modes. The method makes it possible to start from zero speed in closed loop and produce a constant torque at very low speeds by changing the direct vector current component as a function of the speed. The method is implemented and verified experimentally on a 800 W motor. The results demonstrate that the method work successfully over a wide speed range. The method is able to produce estimates of position and speed with a precision good enough to replace a shaft sensor.

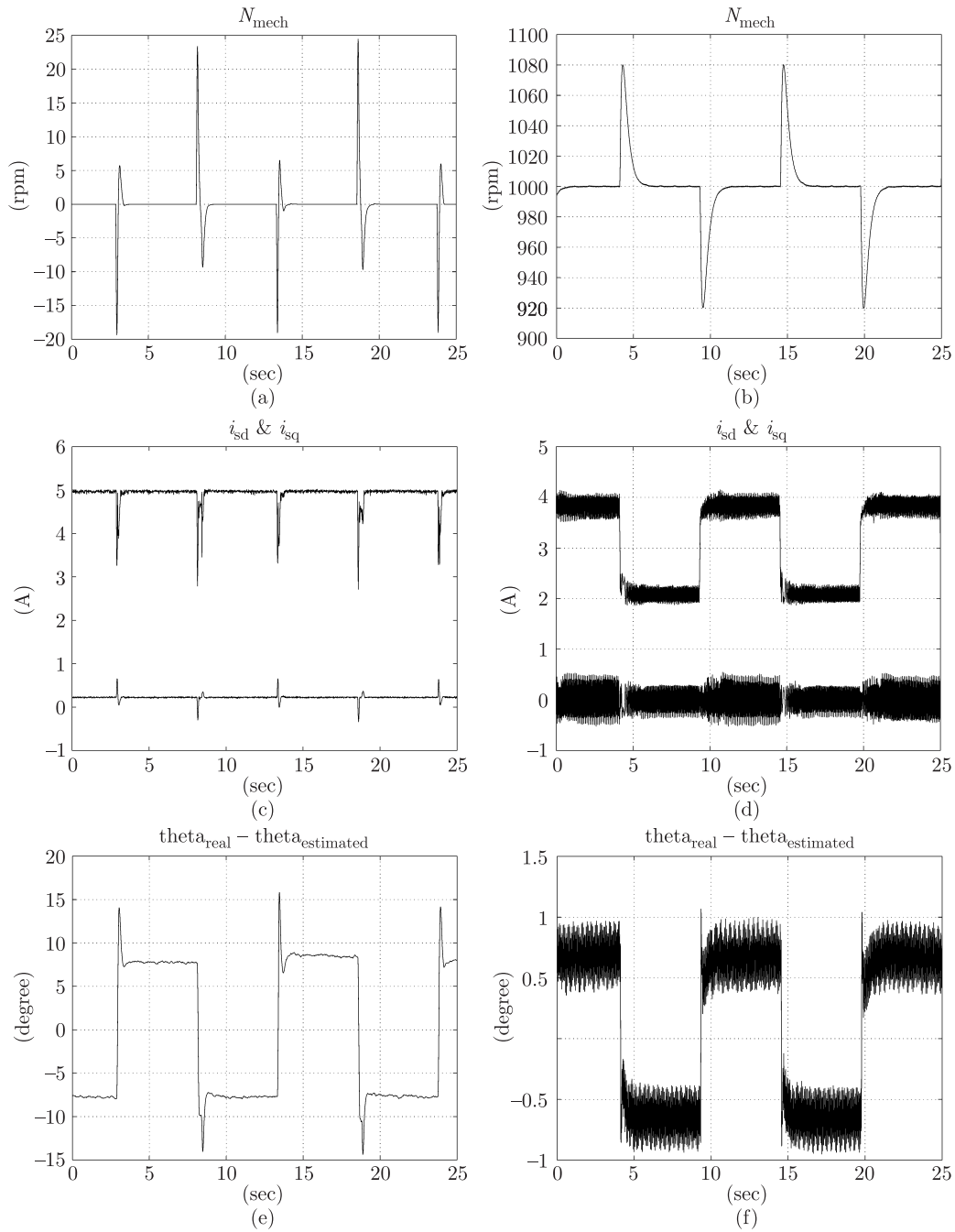


Fig. 5. Torque step at rotor speed 0 rpm (a,c,e) and 1000 rpm (b,d,f), (a) rotor speed  $N_{mech}$ , (b) rotor speed  $N_{mech}$ , (c)  $i_{sd}$  and  $i_{sq}$ , (d)  $i_{sd}$  and  $i_{sq}$ , (e) estimation error  $\hat{\theta} - \theta$  in degrees, (f) estimation error  $\hat{\theta} - \theta$  in degrees

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